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## Appendix E: Measuring the Quantity and Cost of Capital Inputs in Canada

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### E.1 Introduction

**I**N THIS APPENDIX, WE PRESENT THE METHODOLOGY for estimating the indices of capital inputs in Canada over the 1961-98 period. Unlike simple measures of capital stocks, our measure of capital inputs takes into account the changing composition of capital stocks (relatively more equipment than structures). The change in our measure reflects both capital accumulation and a changing composition of capital stocks.

Capital stocks would be a valid measure of capital inputs if capital assets were homogeneous. They are, however, heterogeneous, and their composition changes over time (Griliches and Jorgenson, 1966; and Jorgenson and Griliches, 1967). Tangible assets have different acquisition prices, service lives, depreciation rates, tax treatments, and ultimately different marginal products. The capital stock measure does not account for these differences in capital stocks. However, the capital stock measure is easily available and much work on productivity has used that measure. For example, Statistics Canada employs net capital stock in its productivity estimates at the detailed industry level (Statistics Canada, 1994b). The Organization for Economic Co-operation and Development (OECD) uses gross capital stock in its international comparison of productivity in OECD countries (OECD, 1998).

Jorgenson, Gollop and Fraumeni (1987) constructed indices of capital inputs for forty-six private industrial sectors and the civilian U.S. economy over the 1947-79 period. Their measures of capital inputs incorporate the characteristics of physical assets, cross-classified by six classes of assets (producers' durable equipment, consumers' durable equipment, tenant-occupied residential or non-residential structures, owner-occupied residential structures, inventory, and land) and four legal forms of ownership (corporate business, non-corporate business, household, and institution). More recently, Ho, Jorgenson and Stiroh (1999) extended the analysis and estimated the annual indices of capital inputs for the private U.S. economy over 1948-96.

The desirability of using capital inputs for productivity analysis has been recognized by the U.S. Bureau of Labor Statistics. In 1983, the BLS developed the indices of capital inputs from data on forty-seven types of assets and three major industrial sectors (farm, manufacturing, and other industrial).<sup>1</sup> In recent empirical work, the BLS (1999) made substantial revisions to the procedures used for calculating capital inputs. The most notable are a decrease in the depreciation rates for non-residential structures and a finer classification of capital inputs by industrial sector.

Indices of aggregate capital input have been constructed for Canada by Dougherty (1992), Diewert and Lawrence (1999), and Jorgenson and Yip (1999). Dougherty's indices of capital inputs were built from data on eight types of capital assets and two types of ownership (corporate and personal sectors). Jorgenson and Yip (1999) extended the analysis to a more recent period. Diewert and Lawrence (1999) also constructed indices of aggregate capital input for Canada and examined the sensitivity of their measures to various asset depreciation patterns.

We have constructed indices of capital inputs for the aggregate business sector and each industrial sector in Canada for the 1961-98 period. This appendix explains the methodology and data sources used in the construction of capital inputs. In Section E.2 below, we outline the methodology. In Section E.3, we describe the data sources used for constructing the indices of capital inputs. In Section E.4, we present the annual estimates of capital inputs for each industrial sector and the aggregate business sector for the 1961-98 period. Section E.5 concludes.

## E.2 Methodology for Measuring the Quantity and Cost of Capital Inputs

OUR OBJECTIVE IS TO CONSTRUCT indices of capital inputs or capital services for the business sector and each of the 123 industries of the business sector (called the P-level industry aggregation) over the 1961-98 period. The indices of capital inputs take into account the changing composition of capital stocks and are built from five types of tangible assets: machinery labour equipment, building structures, engineering structures, inventories, and land.

### E.2.1 Estimating Capital Inputs

To construct an index of capital inputs, we assume that the aggregate capital input  $\{K\}$  can be expressed as a translog function of its individual components  $\{K_k\}$ . The growth rate of the aggregate capital input is therefore the weighted average of the growth rates of its components:

$$(1) \quad \Delta \ln K = \sum_k \bar{v}_k \Delta \ln K_k,$$

where  $\Delta$  denotes a first difference, or change between two consecutive periods, for example:

$$(2) \quad \Delta \ln K = \ln K(t) - \ln K(t-1).$$

The weights are given by the average share of the individual components in the value of capital compensation:

$$(3) \quad \bar{v}_k = \frac{1}{2} [v_k(t) + v_k(t-1)], \quad v_k = \frac{c_k K_k}{\sum_k c_k K_k},$$

where  $\{c_k\}$  is the set of user costs of the components of the capital input.<sup>2</sup> At market equilibrium, the user cost of a capital input equals the value of its marginal product. Aggregating capital inputs by means of user costs therefore effectively accounts for the differences in productive contribution from various assets.

The quantity of services for each component of capital input  $\{K_k\}$  is proportional to the stock of capital  $\{A_k\}$  at the beginning of the period:

$$(4) \quad K_k(t) = Q_k A_k(t),$$

where the constants of proportionality  $\{Q_k\}$  transform capital stock into the quantity of services produced by that stock per period.

We assume that the quantity of services delivered per unit of capital stock  $\{Q_k\}$ , per computer for example, is constant at all points in time. The improvement in the quality of computers (e.g., increased processing speed) is incorporated in

the measurement of real capital stock via the proper construction of the price index for computers. Indeed, major efforts have been undertaken in recent years to construct these quality-adjusted price deflators for goods, such as computers that experienced dramatic quality improvement (BLS, 1997; and Gordon, 1997).

Using Equation (4), we can express the growth rate of capital inputs in terms of the growth rates of the capital stock components  $\{A_k\}$ :

$$(5) \quad \Delta \ln K = \sum_k \bar{v}_k \Delta \ln K_k = \sum_k \bar{v}_k \Delta \ln A_k .$$

At the heart of the above methodology for estimating capital inputs is the distinction between capital stock  $\{A_k\}$  and the flow of services received from the capital stock in one period  $\{K_k\}$ . A distinction is also made between the price of acquiring an asset  $\{P_k\}$  and the cost of using the asset for one period.  $\{C_k\}$ . As evident in rental markets, these distinctions exist for computers, automobiles, office equipment and furniture, and so on. In fact, a possible approach to measuring capital inputs would be to compile data on transactions in these rental markets. However, this approach is rarely pursued since there is no rental market for most assets.

The compositional or quality change of capital inputs is the difference between the growth rates of capital inputs and the simple sum of the capital stock components:

$$(6) \quad \Delta \ln Q = \Delta \ln K - \Delta \ln A ,$$

where  $A = \sum_k A_k$  is the simple sum of capital stock components. In Equation (6), the growth of capital inputs ( $\Delta \ln K$ ) is decomposed into capital accumulation ( $\Delta \ln A$ ) and compositional change. ( $\Delta \ln Q$ ) In terms of individual components, the growth rate of capital quality can be written as:

$$(7) \quad \Delta \ln Q = \sum_k \bar{v}_k \Delta \ln A_k - \Delta \ln \sum_k A_k .$$

An examination of Equation (7) shows that capital quality remains unchanged if all components of capital stock increase at the same rate. Capital quality increases if the share of the components with relatively higher user costs (e.g., equipment) increases, and it declines if that share decreases.

### E.2.2 Estimating Capital Stock and the Cost of Capital Services

The indices of capital inputs are constructed using Equation (5) with data on capital stock and the user cost of capital inputs. We assume that assets follow geometric depreciation patterns and calculate capital stock using the perpetual inventory method (see Appendix C for details on the construction of the capital stock). Capital stock of asset  $k$  at the beginning of period  $t$  is:

$$(8) \quad A_k(t) = A_k(t-1)(1 - \delta_k) + I_k(t-1) = \sum_{\tau=1}^{\infty} (1 - \delta_k)^{\tau-1} I_k(t - \tau),$$

where  $I_k$  is real investment of asset type  $k$  and  $\delta_k$  is the depreciation rate.

For an asset with a geometric depreciation pattern, the cost of using the asset over one period or the cost of capital services is (see Jorgenson and Yun, 1991, for details):

$$(9) \quad c_k = \frac{1 - e_k - tz_k}{1 - t} P_k [(r_k - \pi_k) + (1 + \pi_k)\delta_k] + t^p P_k,$$

where  $t$  is the combined federal and provincial corporate income tax rate,  $e_k$  is the investment tax credit,  $z_k$  is the present value of capital cost allowances on one dollar's worth of investment,  $P_k$  is the price of new investment good  $k$ ,  $r_k$  is the nominal rate of return on asset type  $k$ ,  $\pi_k = (P_k(t) - P_k(t-1)) / P_k(t)$  is the capital gain for asset  $k$ ,  $\delta_k$  is the depreciation rate for asset  $k$ , and  $t^p$  is the property tax rate.

The user-cost equation (9) reflects the nominal rate of return on assets, the rate of economic depreciation, and capital gains on assets. It also takes into account the effects of taxation, such as corporate income taxes, investment tax credits, and capital consumption allowances on the user cost of capital.

Since there is no investment tax credit, capital consumption allowances, property tax rate, or economic depreciation, the user-cost equation for land and inventories simplifies to:

$$(10) \quad c_k = \left( \frac{r_k - \pi_k}{1-t} + t_p \right) P_k.$$

All parameters in the user-cost equation are available from various sources. The nominal rate of return on an asset can be estimated in two ways. First, it can be estimated from data on returns to debt and equity. This is problematic due to the multiplicity of returns. In this study, we have chosen to estimate the nominal rate of return *ex post* from data on the total value of capital compensation. We assume that the nominal rate of return is the same for all types of assets in an industry. The nominal rate of return on an asset is chosen such that the sum of the values of the capital input components is equal to the total capital compensation:

$$(11) \quad \sum_k c_k A_k = V, \text{ and } r_k = r,$$

where  $V$  is total capital compensation.

### E.3 Data Sources

THE TWO DATA COMPONENTS used in the construction of capital inputs are capital stock and the cost of capital. Our first task is to construct capital stocks for the five asset types and each industrial sector over the period 1961-98.

The capital stock of depreciable assets (M&E and structures) in the United States was estimated from investment data using geometric depreciation. These U.S. estimates use a 1.65 declining-balance rate for most machinery and equipment, and a 0.9 declining-balance rate for most non-residential structures. Capital stock data published by Statistics Canada are based on a modified double-declining-balance method for both machinery and equipment, and structures. To ensure comparability between Canadian and U.S. capital stock estimates, we obtained an alternative set of capital stock estimates from the Investment and Capital Stock Division of Statistics Canada (see Appendix C). These alternative capital stock estimates have been calculated with the same declining-balance rates as those used in the United States.

These measures will be used in our estimates of capital inputs in Canadian industries. However, for comparison purposes, we also present the results based on capital stocks used in Statistics Canada's productivity estimates.

Inventory stocks are estimated from data in industry balance sheets, national balance sheets, and input-output tables. The industry balance sheets provide data on the book value of inventory stocks at the 1970 three-digit industry classification level for the 1972-87 period. For that period, we set inventory stocks in current prices to their book values. For other years, we have estimated inventory stocks using data on inventory investment from the input-output tables. Since there is no depreciation for inventories, the inventory stock in a year is equal to the stock of the preceding year plus the investment made in the current year. The price deflators of inventory stocks are set to the average of the price deflators of raw materials and final output. Finally, the estimates of inventory stocks are adjusted to the inventory stock of the business sector in national balance sheets.

To estimate land input by industry, we first obtain the nominal value of land in Canada for the 1961-98 period from the National balance sheet accounts. We assume that the quantity of land remains constant and derive its price index. We then remove the real values of farm, residential, and government land from the real value of land in Canada. The remaining non-agricultural, non-government land is allocated across industries. For the 1972-87 period, the allocation is based on the book value of land in the industry balance sheets. For other years, the land value of an industry is extrapolated using the growth of non-residential structures and then adjusted to the national total in National balance sheet accounts.

Our second task is to construct estimates of the user cost by industrial sector for the five asset types over the 1961-98 period.<sup>3</sup> The cost of capital is estimated from data on corporate tax rates, investment tax credit, the present value of capital cost allowances, and economic rates of depreciation.<sup>4</sup> To calculate the combined federal and provincial corporate tax rate of an industry we have taken into account the variation of corporate income tax rates by province, firm size, and the nature of productive activities. First, the corporate income tax rates vary across provinces. While there is only one federal corporate tax rate in all provinces, each province can apply a different corporate income tax rate. Second, Canadian-controlled private corporations (CCPC) are eligible for small business tax reductions. For example, the small business tax rate reduction in 1996 was 16 percent on the first \$200,000 of

active business income. Third, since 1973, a tax reduction is offered to corporations engaged in manufacturing and processing. As an example, the manufacturing and processing tax credit after 1994 is 7 percent of federal tax on manufacturing and processing profits that do not qualify for the small business deduction. Table E.1 presents the combined federal and provincial tax rates in 1996 by province, firm size, and type of productive activities.

The average corporate tax rate of an industry is calculated as a weighted-sum of statutory tax rates using appropriate taxable income shares as weights. Data on taxable income by province, firm size and industry are obtained from the industry balance sheets and income statements. From these sources, we have the distribution of taxable income across the ten provinces by industry for the 1961-87 and 1993-96 periods, and the share of small business deduction by industry for the 1974-94 period. The income shares for other years are set to the shares of the nearest year.

To encourage investment, a credit was granted for new production facilities as of 1976. Starting at 5 percent for all industries, the rate was raised to 7 percent in 1979 and regional variations with higher rates were introduced. In 1989, investment tax credits were discontinued except for the Atlantic provinces (Williamson and Lahmer, 1996).

<b>Table E.1</b>				
<b>Combined Federal and Provincial Corporate Tax Rate, 1996</b>				
<b>Province</b>	<b>General Rate</b>	<b>Small Business Rate</b>	<b>Small Business M&amp;P Rate</b>	<b>M&amp;P Rate</b>
Newfoundland	43.12	18.12	18.12	27.12
Prince Edward Island	44.12	20.62	20.62	37.12
Nova Scotia	45.12	18.12	18.12	38.12
New Brunswick	46.12	20.12	20.12	39.12
Quebec	45.37	18.87	18.87	31.02
Ontario	44.62	22.62	22.62	35.62
Manitoba	46.12	23.12	23.12	39.12
Saskatchewan	46.12	21.12	21.12	39.12
Alberta	44.62	19.12	19.12	36.62
British Columbia	45.62	22.12	22.12	38.62

Source: Williamson and Lahmer, 1996.

For a declining-balance depreciation method, the present value of depreciation allowances in the user-cost equation is (for details, see Dougherty, 1992):

$$(12) \quad z = t\alpha / [i(1-t) + \alpha],$$

and the present value of depreciation allowances for a straight-line depreciation is:

$$(13) \quad z = \frac{\mu\alpha [1 + i(1-t)]}{i(1-t)} \left( 1 - \frac{1}{[1 + i(1-t)]^T} \right),$$

where  $\alpha$  is the tax-allowable depreciation rate,  $i$  is the nominal interest rate on the Government of Canada three-month treasury bills (Cansim matrix 2560, B140007), and  $T$  is the lifetime of the asset.

To calculate the present value of capital cost allowances, we take geometric depreciation rates of 5 percent for structures and 20 percent for equipment over the entire period with the following exceptions. First, the straight-line method was used for structures and equipment for the 1961-66 period. During that period, equipment was written off within 2 years ( $T = 2$ ,  $\alpha = 0.5$ ) and structures within 5 years ( $T = 5$ ,  $\alpha = 0.2$ ). Second, before 1981, firms were able to claim a full year's capital cost allowance on an asset in the year it was acquired. After 1981, only one-half of the normal capital cost allowance was written off in the year an asset was acquired, the remainder being depreciated over subsequent years. Third, after 1972, accelerated capital cost allowances were granted for machinery and equipment used in manufacturing and processing (M&P) activities. Equipment for M&P firms was written off within two years during the 1972-81 period and within three years after 1981.

The economic rate of depreciation is set to be the rate implicit in capital stock and investment data. It is equal to gross investment in a year minus net investment in the year, divided by capital stock at the beginning of the year:

$$(14) \quad K(t+1) = K(t)[1 - \delta(t)] + I(t), \text{ or } \delta(t) = \frac{I(t) - [K(t+1) - K(t)]}{K(t)}.$$

Business property taxes in the user-cost equation are mainly levied on land and structures, with machinery being free of such taxes. To estimate the property tax rates, we first obtain the property tax base as the nominal values

of land and structures at the P-level of industry aggregation. We then divide the tax base into taxes on production from the input-output tables to get average property tax rates.

## E.4 Empirical Results

THIS SECTION PRESENTS THE INDICES OF CAPITAL INPUTS in each industrial sector and the aggregate business sector for the 1961-98 period. The indices of capital inputs are aggregated from five asset types: machinery labour equipment (M&E), building structures, engineering structures, land, and inventories.

Figure E.1 shows the share of fixed reproducible investment by asset type.<sup>5</sup> The share of M&E in real fixed reproducible investment has grown steadily over the period 1961-98. The investment share of equipment almost doubled over that period, from 34 percent in 1961 to 60 percent in 1998. The increase in the share of equipment occurred at the expense of structures. The share of building structures fell from about a quarter of total investment in 1961 to 13 percent in 1998. Similarly, the share of engineering structures fell from about 40 percent to little over a quarter of total investment over that period.

These investment patterns directly determine the composition of the capital stock. The rising share of M&E in total investment leads to an increase in its capital stock share, as shown in Figure E.2. The share of equipment in total capital has experienced the fastest growth. It increased from 13 percent in 1961 to 22 percent in 1998. The structures' share of total capital also increased from 1961 to 1998, while the share of inventories remained virtually unchanged. The land share declined sharply over the period – from 37 percent of total capital stock in 1961 to a little over 10 percent of capital stock in 1998. Land was the largest component of total capital stock in 1961. However, its share had fallen below the capital shares of M&E, non-residential building structures, and engineering structures in 1998. This shift in the composition of capital stock towards short-lived equipment is a major source of the increase in capital quality.

All else being equal, short-lived equipment has a higher depreciation rate and thus a relatively high user-cost. This is evident from Figure E.3, which shows the highest user cost of equipment for almost all years during the 1961-95 period. Land input has the lowest user cost for most of the period. Compared with engineering structures, the user cost of equipment and

building structures declined over the 1961-95 period. The decline in the relative user cost of equipment is mainly attributed to the decline in the price of equipment relative to engineering structures. Figure E.3 also shows that the user cost of capital exhibited large fluctuations over the business cycle.

Figure E.4 shows the indices of capital inputs, capital stock and capital quality in the business sector.<sup>6</sup> The indices of capital services, capital stock, and capital quality all increased over the 1961-98 period. Capital services growth was faster than capital stock growth, partly reflecting the substitution of relatively short-lived and high user-cost equipment for long-lived and low user-cost structures. This shift in the capital stock composition directly lead to an increase in capital quality over the period.

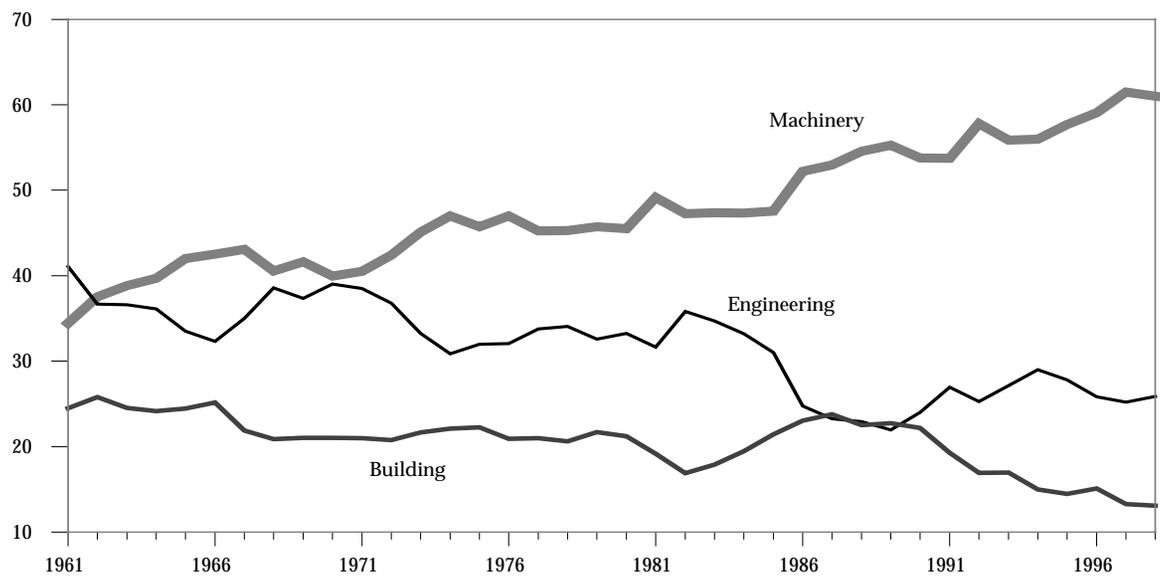
Table E.2 provides a decomposition of capital input growth in the business sector for the period 1961-98 and three sub-periods: 1961-73, 1973-88, and 1988-98. Capital inputs increased at an annual rate of 3.11 percent in the business sector during the 1961-98 period. Of the 3.11 percent growth in capital inputs, capital accumulation contributed 2.63 percentage points while the compositional or quality change contributed 0.48 percentage points.

The slow growth during the 1990s was the most noticeable trend of capital inputs in the business sector. The index of capital inputs increased at 2.21 percent per year over the 1988-98 period, compared to 3.65 percent over the 1961-73 period and 3.28 percent over the 1973-88 period.

We have also constructed indices of capital inputs, capital stock, and capital quality for each industrial sector over the 1961-98 period (see Chapter 3). Our estimates show that capital input growth exceeded capital stock growth and capital quality increased in a majority of industries over that period.

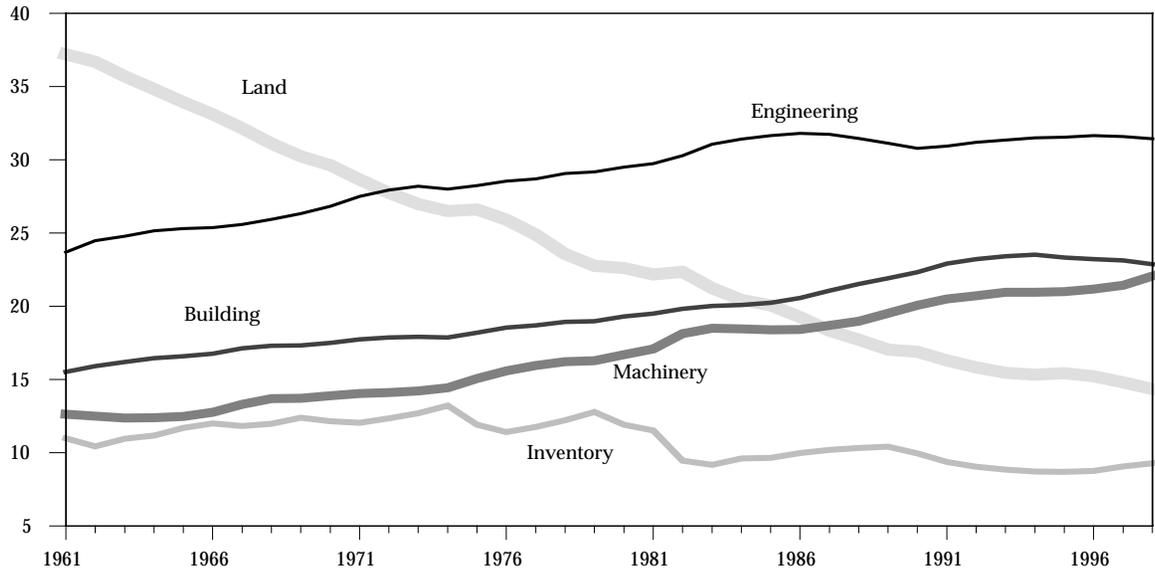
	<b>1961-98</b>	<b>1961-73</b>	<b>1973-88</b>	<b>1988-98</b>
Growth of Capital Inputs	3.11	3.65	3.28	2.21
Growth of Capital Stock	2.63	2.98	2.85	1.88
Growth of Capital Quality	0.48	0.67	0.43	0.32

**Figure E.1**  
**Share of Investment by Asset Type (%)\***



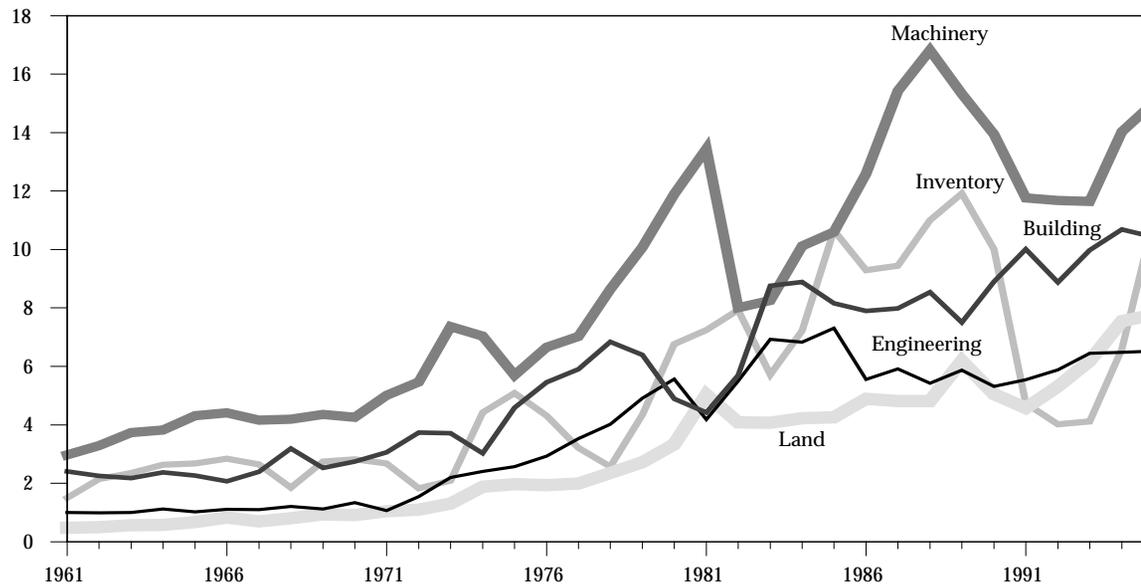
\* Investment in constant 1992 dollars.

**Figure E.2**  
**Share of Capital Stock by Asset Type (%)\***



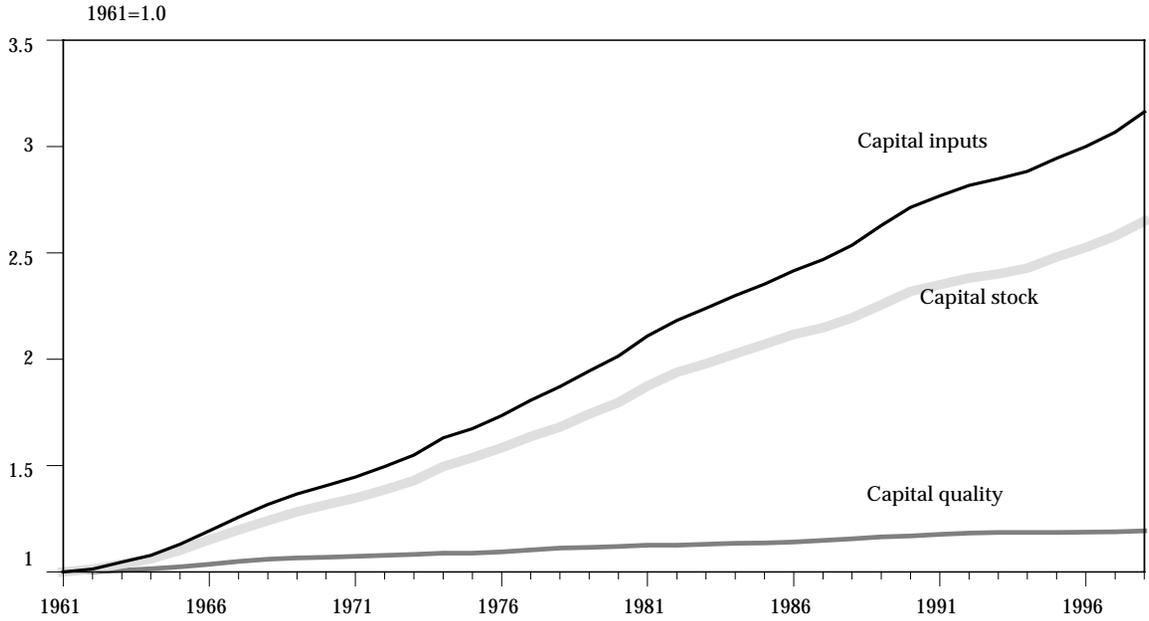
\* Capital stock in constant 1992 dollars.

**Figure E.3**  
**User Cost by Asset Type (%)\***



\* The user cost is set to 1.0 in 1961 for engineering structures.

**Figure E.4**  
**Indices of Capital Inputs, Capital Stock and Capital Quality**



	<b>1961-98</b>	<b>1961-73</b>	<b>1973-88</b>	<b>1988-98</b>
Growth of Capital Inputs	2.71	3.47	2.90	1.52
Growth of Capital Stock	1.85	2.40	1.98	1.01
Growth of Capital Quality	0.86	1.07	0.92	0.52

The capital input measures presented above are constructed from capital stocks estimated using the BEA methodology. M&E capital stock is estimated using a 1.65 declining-balance depreciation rate and structure capital stock is estimated using a 0.91 declining-balance rate. For a comparison, we have also constructed the indices of capital inputs using capital stock data from Statistics Canada's KLEMS database. These capital stocks are estimated using a modified double-declining-balance method (see Appendix C for details). The results are presented in Table E.3. A comparison of Tables E.2 and E.3 shows that capital stock in the KLEMS database grew much more slowly than the capital stock estimated with the U.S. methodology. The growth of capital inputs aggregated from the KLEMS capital stock data was also slower.

## E.5 Conclusion

THIS APPENDIX PRESENTS THE METHODOLOGY for estimating the indices of capital inputs and capital quality for each industrial sector and the aggregate business sector over the 1961-98 period. We find that capital inputs, capital stock, and capital quality all increased in the business sector during that period. A decomposition of capital input growth shows that both capital accumulation and the change in composition contributed to the growth of capital inputs in the business sector. The increase in capital inputs and capital quality was pervasive across industries during the 1961-98 period.

A noticeable trend in capital inputs was the slow growth during the 1990s. The annual growth rate of capital inputs in the 1990s was slower than during the preceding three decades by over a full percentage point. This occurred despite the dramatic increase in investments in information and communications technology in the 1990s.

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## Notes

- 1 But it was not until 1993 that the BLS developed analogous measures of labour input that incorporate characteristics of workers such as age, sex, and education.
- 2 A distinction is sometime made between the user cost of capital and the rental price of capital (Jorgenson and Yun, 1991).  $c_k$  in Equation (3) is usually called the rental price of capital services and it measures the unit cost of using a capital good for a specified period of time. The cost of capital is defined as the rental price of capital services divided by the price of acquisition of a capital good. The cost of capital therefore transforms the acquisition price of capital good into rental price. In this paper, we will use the rental price and the user cost of capital interchangeably. They both measure the unit cost of using a capital good for a specified period of time.
- 3 There were no estimates of sectoral capital compensation after 1995 at the time of this study. The cost of capital over the 1996-98 period is assumed to be the same as in 1995 for the purpose of constructing the capital inputs.
- 4 The cost of capital in this study does not include every provision in the corporate tax system that could have an impact on the cost of using capital services in one specified period. For example, the estimates of the user cost do not take into account the special treatment of banks and insurance companies, the provisions for intangible exploration and drilling costs, and depletion allowances in resource extraction industries. To the extent these special tax treatments have the same proportional effect on the cost of using all types of assets within a given industry, they will not affect the indices of capital services and capital quality for that industry.
- 5 Real land stock is assumed to be constant in the aggregate business sector and there is no investment in land. Inventory investment is very sensitive to cyclical fluctuations. Therefore, we only present the composition of fixed reproducible investment.
- 6 For the purpose of constructing the capital inputs, the user cost of an asset over the 1996-98 period is assumed to be the same as in 1995.

